江蘇科技大學
Jiangsu University of Science and Technology

Hydrodynamic Analysis on Ships and Offshore Structures

Jiangsu university of science and technology

Teaching contents

1 2.4 Linear wave radiation by floating structures

Radiation? Diffraction?

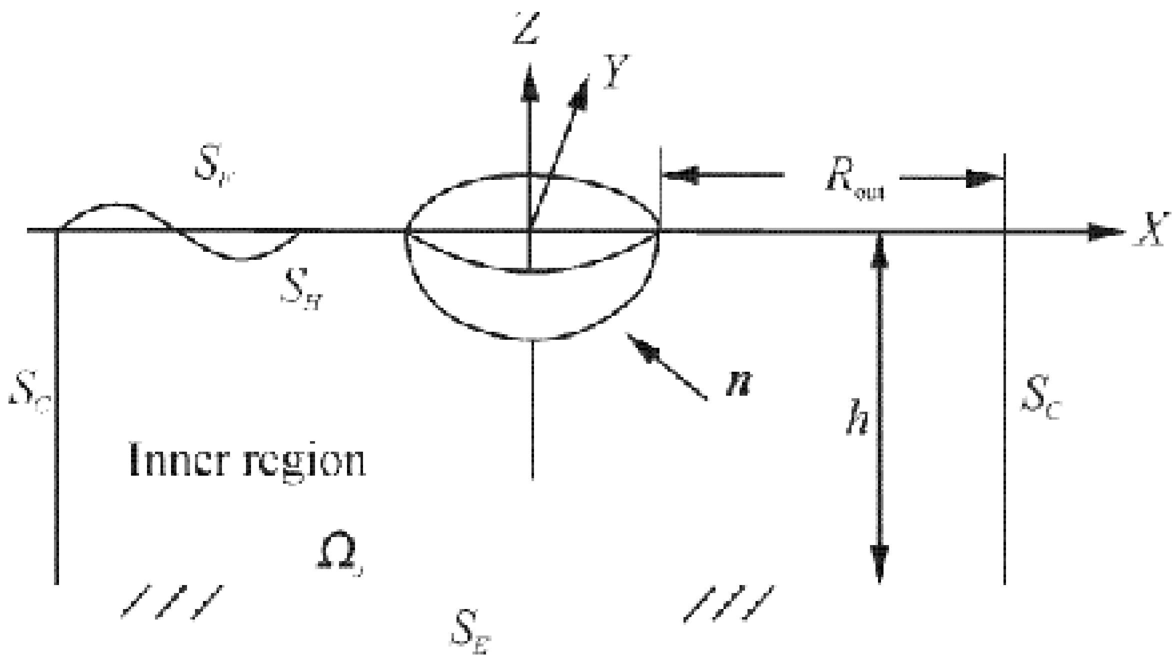
Radiation, only has 3D body (added mass…).

Diffraction, body is fixed, which is wave induced motion.

Mathematical formulation

With the assumptions that an arbitrary floating body on the free surface oscillates with small amplitude compared to a principle body dimension and the water depth is not small compared with a typical wave length of the radiated wave, it is possible to apply Taylor series expansions to transform the body surface condition at the mean body surface and transform the free surface boundary condition at the still water surface. Using Stokes expansion procedure, the first-order quantities can be separated by introducing perturbation series

Mathematical formulation



Definition sketch

2.4、Linear wave radiation by floating structures

Mathematical formulation

The reference system of Cartesian coordinates is defined by letting (x, y) plane coincide with the mean free surface and z points vertically upward from the still water level as shown in Fig.1. The body surface is denoted by S_H and its unit normal vector directed outward from the fluid region is denoted by $\boldsymbol{n}$. The seabed S_B is assumed horizontal along the plane $z = h$. Let t denote time and η be the free surface elevation relative to the still water surface S_F. An artificial boundary S_C is introduced as shown in Fig.1, which divides the fluid domain into inner region and out region. The boundary S of inner region is $S = S_F \cup S_H \cup S_B \cup S_C$.

Mathematical formulation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \qquad \text{in } \Omega_f \qquad (1)$$

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0, \qquad \text{on } S_F; \qquad (2)$$

$$\frac{\partial\phi}{\partial \boldsymbol{n}} = V_n(t), \qquad \text{on } S_H; \qquad (3)$$

$$\frac{\partial\phi}{\partial \boldsymbol{n}} = 0, \qquad \text{on } S_B; \qquad (4)$$

where g is the acceleration due to gravity, and V is the velocity of the body.

For simple case, S_c can not be penetrated.

Mathematical formulation

The Integration form of free-surface boundary condition (IFBC)

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial z} = 0 , \qquad \text{on } S_F;$$

$$\int_0^{\tau} \mathrm{d}\tau_1 \int_0^{\tau_1} f(t)\mathrm{d}t = \int_0^{\tau} f(t)\mathrm{d}t \int_t^{\tau} \mathrm{d}\tau_1 = \int_0^{\tau} (\tau - t) f(t)\mathrm{d}t,$$

$$\phi(p,t) = -g\int_0^{t} (t-\tau)\frac{\partial \phi(p,\tau)}{\partial z}\mathrm{d}\tau.$$

Mathematical formulation

three dimensional green theorem

$$\phi(p,\tau)=\frac{1}{2\pi}\iint_S\left[G(p,q)\frac{\partial\phi(q,\tau)}{\partial \boldsymbol{n}_q}\right]\mathrm{d}s_q-$$

$$\frac{1}{2\pi}\iint_S\left[\phi(q,\tau)\frac{\partial}{\partial \boldsymbol{n}_q}G(p,q)\right]\mathrm{d}s_q.$$

Mathematical formulation

three dimensional green theorem

Here $p(x,y,z)$ is a field point and $q(\xi,\eta,\zeta)$ is a source point on the surface of the domain, $G(p,q)$ is a Green's function. For cases in which the seabed is horizontal, a Green's function which contains the fundamental solution of the Laplace equation and its images can be chosen to account for the symmetry about the seabed. In this manner, S_B can be excluded from the surface of the domain, and the mesh on the seabed does not need to be generated. This Green's function is

$$G(p,(q,q')) = \sum_{k-1}^{2} \frac{1}{r_k},$$

Mathematical formulation

three dimensional green theorem

$$G(p,(q,q')) = \sum_{k-1}^{2} \frac{1}{r_k},$$

where r_k is the distance between the field point $p(x,y,z)$ and source point $q(\xi,\eta,\zeta)$ and $q'(\xi,\eta,(2h+\zeta))$. q' is the image of q about the seabed. Thus, r_k is given by

$$r_1 = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2},$$

$$r_2 = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+2h+\zeta)^2}.$$

Mathematical formulation

three dimensional green theorem

$$\sum_{j=1}^{nS_H} D_{ij}\phi_{jm} - \sum_{j=nS_H+1}^{nS_H+nS_F} S_{ij}\left(\frac{\partial\phi}{\partial n}\right)_{jm} - \sum_{j=nS_H+nS_F+1}^{nS_H+nS_F+nS_C} S_{ij}\left(\frac{\partial\phi}{\partial n}\right)_{jm} =$$

$$\sum_{j=1}^{nS_H} S_{ij}(V_n)_{jm} - \sum_{j=nS_H+1}^{nS_H+nS_F} D_{ij}\phi'_{jm} - \sum_{j=nS_H+nS_F+1}^{nS_H+nS_F+nS_C} D_{ij}\phi_{jm},$$

$i=1, 2, \cdots, \quad (nS_H + nS_F + nS_C)$

$$\phi'_{jm} = -g(\Delta t)^2 \sum_{k-1}^{m-1}(m-k)\left(\frac{\partial\phi}{\partial n}\right)_{jk}, \qquad \text{on } S_F$$

Mathematical formulation

three dimensional green theorem

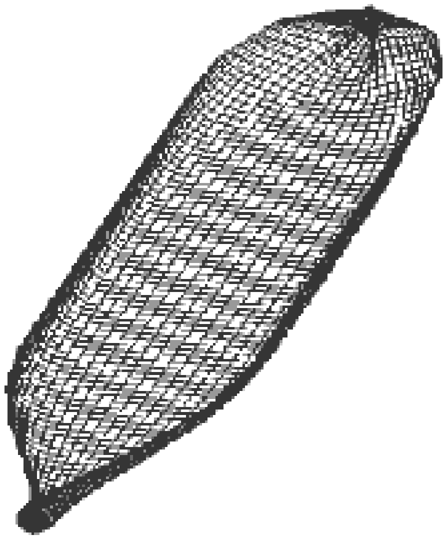
The matrix coefficients S_{ij} and D_{ij} correspond to integrals of the Green's function and its normal derivative over the area ΔQ of the jth facet respectively. S_{ij} and D_{ij} are written as

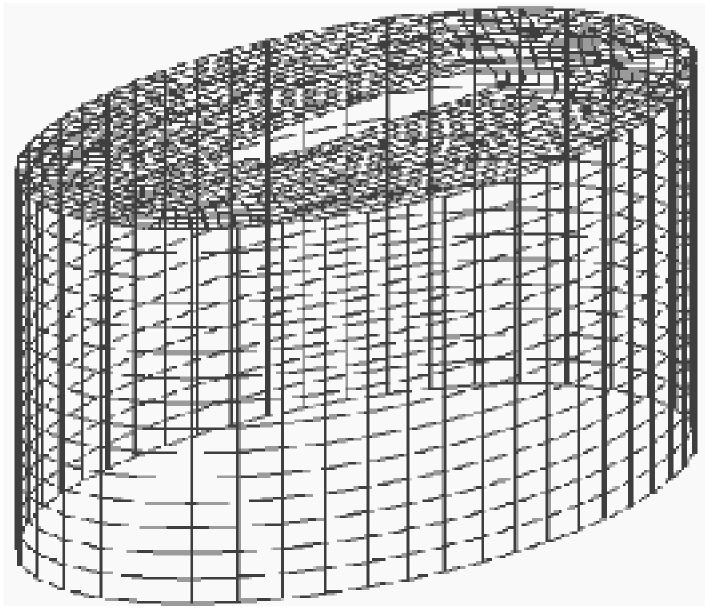
$$S_{ij} = \int_{\Delta Q_j} G_{ij} \mathrm{d}s_j,$$

$$D_{ij} = \begin{cases} \int_{\Delta Q_j} \frac{\partial G_{ij}}{\partial \boldsymbol{n}_j} \mathrm{d}s_j, & i \neq j; \\ 2\pi, & i = j. \end{cases}$$

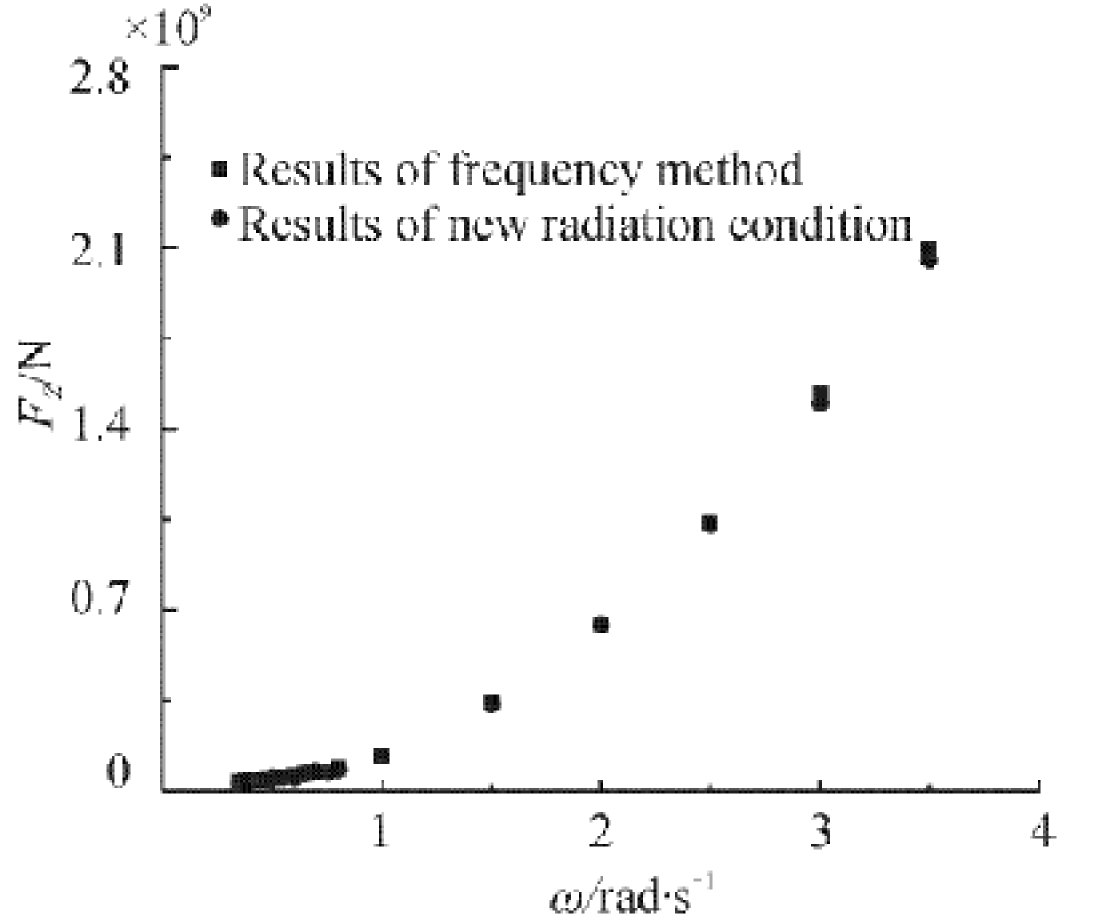
Results

Numerical simulation of liquefied natural gas carrier with elliptic plan-form domain



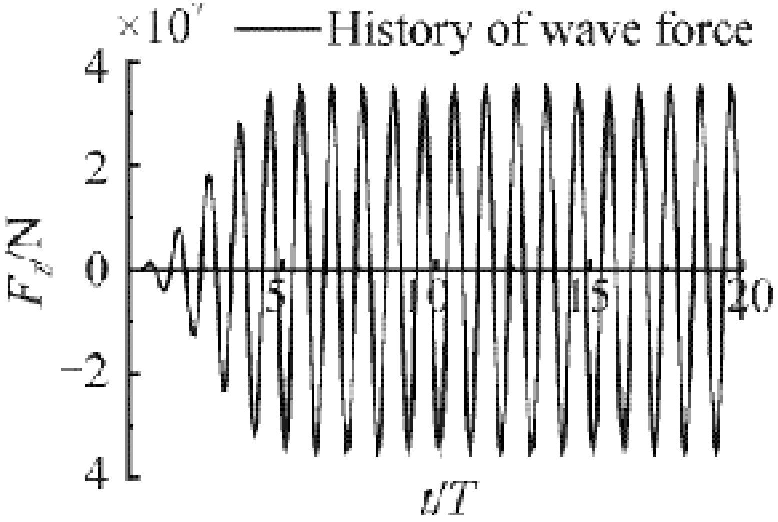


Results

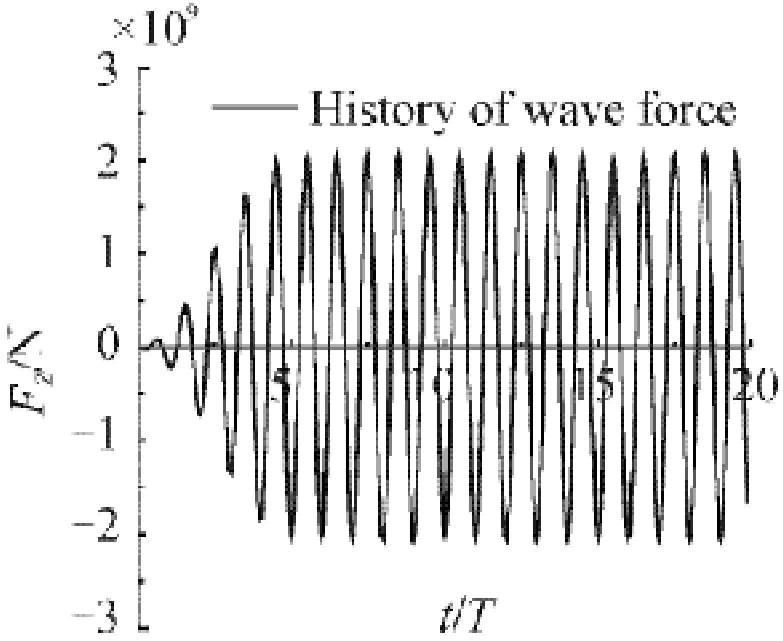


Comparison of F_Z with frequency solution at different ω

Results



(a) ω=0.35



(e) ω=3.5

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Thank you！

School of naval architecture and ocean engineering